

# Laser fields and proxy fields

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(28 September 2016)

## Abstract

The convention in Atomic, Molecular, and Optical (AMO) physics of employing the dipole approximation to describe laser-induced processes replaces the four source-free Maxwell equations governing laser fields with a single Maxwell equation for a “proxy” field that requires a virtual source current for its existence. There is no possible gauge equivalence between these fields. The proxy field is serviceable for some purposes, but its applicability is limited, and the qualitative models it evokes can be inappropriate or erroneous. One example is the “above-threshold ionization” (ATI) phenomenon; surprising for proxy fields, but natural and predicted in advance of observation for laser fields. A serious problem occurs as the field frequency declines; the proxy field approaches a constant electric field, in contrast to laser fields that propagate with the velocity of light for all frequencies, with increasing low-frequency importance of the magnetic component. An often-overlooked limitation is that numerical solution of the Schrödinger equation is exact for proxy fields, but not for laser fields. An important corollary shown here is that nondipole corrections to the proxy field cannot produce equivalence to a laser field because of basic differences in the Maxwell equations.

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The study of strong-field laser interactions with matter has been divided into relativistic and nonrelativistic phenomena, with the dipole approximation conventionally employed in the nonrelativistic case. The investigation carried out here begins with a comparison of the Maxwell equations governing actual laser fields with those relating to the dipole approximation. These equations are so different, mathematically and in their physical significance, that the terminology is adopted of referring to the dipole-approximate version as a *proxy field* employed to represent an actual laser field. The task is then to examine how well the proxy field mimics the actual laser field. The ability of the proxy field to match successfully many laboratory phenomena means that the mimicry can be quite good in those cases, but it is also found that the mimicry is not merely bounded in extent but can be seriously in error in important ways. A basic difficulty with the proxy field is that it cannot exist without a postulated virtual source current. This contrasts with laser fields that, after their initial creation, propagate in vacuum indefinitely without sources to sustain them. The virtual source current is necessary for mimicry, but it can also introduce unwanted and unexpected consequences, one of which is the unphysical insertion of external energy and momentum if the virtual source current is employed incautiously. Some inappropriate results of faulty mimicry have been accepted as valid in the strong-field community.

A fundamental difficulty that has yet to be addressed is that the defects of the dipole approximation cannot be resolved by simply adding nondipole corrections to it. That is one of the most important conclusions of this investigation.

The vacuum Maxwell equations in Gaussian units for the electric field  $\mathbf{E}$  and the magnetic field  $\mathbf{B}$  are

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2}$$

$$\nabla \times \mathbf{B} - \frac{1}{c}\partial_t\mathbf{E} = \frac{4\pi}{c}\mathbf{J}, \tag{3}$$

$$\nabla \times \mathbf{E} + \frac{1}{c}\partial_t\mathbf{B} = 0, \tag{4}$$

where  $\rho$  and  $\mathbf{J}$  are the source charge and current densities. As applicable to any transverse field such as that of a laser, these equations apply with no source terms, so that laser fields

are governed by the equations

$$\nabla \cdot \mathbf{E} = 0, \quad (5)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (6)$$

$$\nabla \times \mathbf{B} - \frac{1}{c} \partial_t \mathbf{E} = 0, \quad (7)$$

$$\nabla \times \mathbf{E} + \frac{1}{c} \partial_t \mathbf{B} = 0. \quad (8)$$

The symmetry between electric and magnetic fields that is evident in these equations is a hallmark of transverse fields. An important reminder is that, in any laser experiment, the only fields that can actually reach the target are propagating fields. (The descriptions *transverse*, *propagating*, and *plane-wave* are used interchangeably here.) Any extraneous fields introduced by interactions with optical elements cannot persist beyond a few wavelengths from those disturbances. Thus, the beam reaching the target can only consist of fields that obey the plane-wave equations (5) to (8).

The dipole approximation (DA), as conventionally employed in the Atomic, Molecular, and Optical (AMO) community, is defined by the substitutions

$$\mathbf{E}(t, \mathbf{r}) \rightarrow \mathbf{E}^{DA}(t), \quad \mathbf{B}(t, \mathbf{r}) \rightarrow \mathbf{B}^{DA} = 0. \quad (9)$$

The applicable Maxwell equations follow from zero values for all expressions in (1) - (4) containing the  $\nabla$  operator, and the complete absence of  $\mathbf{B}$ . With these limitations, the only surviving equation is (3), which can be written as

$$\partial_t \mathbf{E}^{DA}(t) = -4\pi \mathbf{J}^{DA}(t), \quad (10)$$

where  $\mathbf{J}^{DA}(t)$  is a virtual source current that must be posited in order to satisfy the Maxwell equation applicable to the dipole approximation. The fidelity of dipole-approximation mimicry to actual laser fields is determined by how well the effects of the proxy field in Eq. (10) replicate those of laser fields in Eqs. (5) - (8).

An important example of effective mimicry is the matter of *rescattering*, a process in which an electron ionized from an atom follows a path that returns it to the ion, upon which it can recombine with the accompanying emission of higher harmonics of the laser field. The path of a free electron in the laser field [1] matches, in the nonrelativistic case, exactly that of the electron driven by the virtual source  $\mathbf{J}^{DA}$  [2, 3], as long as the comparison is

limited to not much more than a wavelength in space or a wave period in time. Beyond those narrow limits, the virtual source current injects unphysical energy into the problem, resulting in a photoelectron trajectory that fails to mimic the motion of a free electron in the laser field. The rescattering case is especially instructive because it illustrates that effective mimicry evokes models that are completely different from those appropriate to laser fields. The proxy field forces an oscillatory electron motion, called a *quiver motion*, with a kinetic energy termed a *ponderomotive energy*. In the free electron case, the same magnitude of ponderomotive energy is involved, but it is not kinetic; it arises from a “mass shift” of a charged particle in a plane-wave field [4].

One fact immediately evident from comparing the Maxwell equations (5) - (8) for a laser field with (10) for the proxy field is that there can be no gauge equivalence connecting them. Gauge transformations preserve the fields, and it is obvious that the laser field and the proxy field are not the same. This has consequences because negative conclusions have been reached about the lone analytical approximation theory for laser effects [5–7] that is based on (5) - (8), by showing the differences between its predictions and those following from tunneling theories [8–11]. All tunneling theories are based on the dipole-approximation Maxwell equation (10). The verdict that the proxy theories are superior [12] comes from comparing the analytical approximations with numerical solutions of the Time-Dependent Schrödinger Equation (TDSE). The TDSE result is considered to be exact, although it is actually based on the proxy-field theory of (10). The reasoning is circular; all that is shown is that analytical approximations arising from the DA agree with numerical solution of the Schrödinger equation arising from the DA. The fact that these results disagree with the laser-field results of Ref. [5] is a judgment to the detriment of the proxy field, rather than in favor of the proxy theory as concluded in Ref. [12].

Clarifying remarks about the 1980 theory [5] are appropriate here. A completely relativistic theory of strong-field ionization was developed in 1990 [7], based on the Volkov solution, which is an exact solution of the Dirac equation for the behavior of a charged particle in a laser field. When the nonrelativistic limit of this theory is taken, the result so obtained is the 1980 theory [5–7, 13]. The 1980 theory thus arises directly from Eqs. (5) - (8). It includes the effects of the magnetic component of the laser field, since the relativistic theory retains the full  $(\omega t - \mathbf{k} \cdot \mathbf{r})$  phase, and performs integrations over spatial coordinates before the nonrelativistic limit is taken. This makes the 1980 theory applicable in that middle

ground of  $v/c$  effects (magnetic field effects) even when the fully relativistic  $(v/c)^2$  effects have been neglected. This capability has already been demonstrated in the success of the 1980 theory in explaining [14] the energetic double-peak spectrum found at  $10\mu m$  with a  $CO_2$  laser [15], something completely inexplicable with a DA theory.

Because the final results of the 1980 theory bear a superficial resemblance to proxy-field theories, the genesis of the 1980 theory has not been duly noted. The evidence for this is the persistence of the self-contradictory terminology “KFR”, where “K” refers to the tunneling theory of Keldysh [8] based on the proxy-field of Eq. (10), and “R” refers to the 1980 theory [5] based on the laser-field Eqs. (5) - (8). The terminology Strong-Field Approximation (SFA) was introduced in 1990 [6] to distinguish the laser-field theory from proxy-field theories, but this attempt at clarity was nullified by the adoption in 1994 [16] of “SFA” to label all analytical approximations.

A fundamental problem arising from the contrasting Maxwell equations for laser fields and proxy fields is the matter of nondipole corrections. It has been pointed out [17] that the well-known failure of the dipole approximation at very high frequencies is overshadowed by the little-known but very important failure of the dipole approximation at low frequencies, brought on by the increasing importance of the magnetic component of a laser field as the frequency declines. It has been suggested by specialists in TDSE calculations that an apparently straightforward solution of the problem is to incorporate nondipole corrections into the proxy-field Schrödinger equation. However, no alteration of Eq. (10) can replicate Eqs. (5) - (8). The result of nondipole corrections in a theory based on the proxy Maxwell equation (10) would serve only to provide information about longitudinal waves [18], a little-known phenomenon unrelated to laser fields.

When a laser field, characterized by its unique ability to propagate indefinitely without the benefit of charge or current sources, is modeled by a proxy field completely devoid of a magnetic component and dependent for its existence on a virtual current, it is inevitable that qualitative understanding of laser-induced phenomena can be importantly different in the two cases. One instance of this has already been noted in the matter of the ponderomotive energy, which is a kinetic energy for the proxy field and more in the nature of a potential energy for the laser field. Another example of basic importance is the Above-Threshold Ionization (ATI) phenomenon [19], wherein the long-familiar dominance of the lowest-order process in perturbation theory is replaced by a concept in which many orders can contribute,

and where the lowest order might not even be the most important. The proxy theory, with its dependence on a single Maxwell equation, appears to be governed by a single parameter, the Keldysh parameter  $\gamma_K$ , defined as

$$\gamma_K = \sqrt{E_B/2U_p}, \quad (11)$$

where  $E_B$  is the energy by which an electron is bound in an atom, and  $U_p$  is the already-discussed ponderomotive energy of the free electron in the field. This leads to a physical model divided into two domains, where the low-frequency *tunneling* domain ( $\gamma_K < 1$ ) shows no clearly distinguished peaks in photoelectron spectra, and the higher-frequency *multiphoton* domain ( $\gamma_K > 1$ ) reveals individual peaks in a spectrum that are identifiable with specific numbers of photons that participate in the process. Tunneling is an explicitly proxy-field process since it comes about through the interference of two longitudinal fields corresponding to the Coulomb binding potential and the scalar-field proxy described by Eq. (10). There is no such concept as tunneling in a laser-field theory, since a laser field is a vector field that has no possible gauge connection to a scalar field such as can be represented by a scalar potential such as  $\mathbf{r} \cdot \mathbf{E}(t)$ . Two separate intensity parameters are required for the laser-field theory, conveniently expressed in terms of the ratio of the basic ponderomotive energy to the energy of a photon and to the binding energy in the atom, and expressed as [5]

$$z = U_p/\hbar\omega, \quad z_1 = 2U_p/E_B. \quad (12)$$

The  $z_1$  parameter is related to the Keldysh parameter ( $z_1 = 1/\gamma_K^2$ ), but nothing equivalent to  $z$  exists for the proxy field.

These manifest differences in the basic descriptions of laser fields and proxy fields are related to the physical interpretation of the ATI phenomenon. The first observation of ATI [19] was regarded as a shocking and unexpected development within the DA-dominated AMO community, but it was obvious and predicted in full detail in advance of the laboratory observation within a laser-field treatment. In fact, important aspects of the ATI phenomenon that were not observed until 1986 [20] were already predicted by the theory published in 1980 [5]. The 1980 theory was created and discussed prior to 1979. (See also Ref. [21], where some ATI features are described.) That is, ATI was predicted in advance of its initial observation. This was possible because a theory of transverse fields was employed.

Figure 1 is shown here for several reasons. The experimental data plotted in the figure are from the 1986 experiments by Bucksbaum, *et al.* [20]. A 1987 paper [22] based on

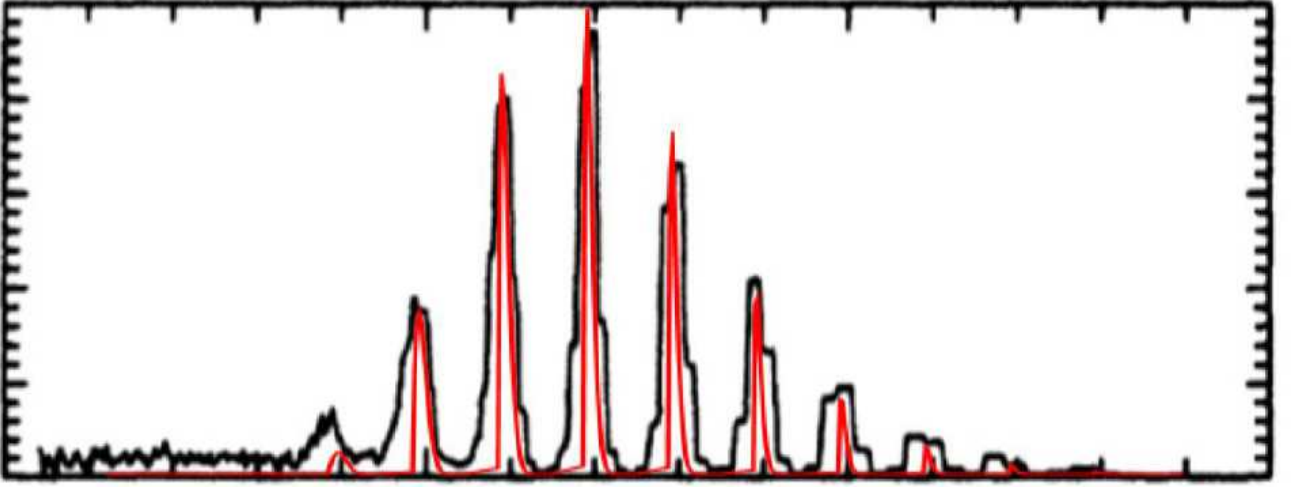


FIG. 1: (color online) This figure shows the ability of the transverse-field theory of Ref. [5] to replicate the experimental results presented in Ref. [20]. The black curve (with wide peaks) is the measured photoelectron spectrum and the red curve (with narrow peaks) is the theoretical fit. Laser parameters:  $1064nm$ , peak intensity  $2 \times 10^{13}W/cm^2$ , pulse duration  $100ps$  ( $z = 1.82$ ,  $z_1 = 0.35$ ,  $\gamma_k = 1.69$ ) on a xenon target. The calculation includes focal averaging in a Gaussian beam with Gaussian time distribution, and with partial ponderomotive energy ( $U_p$ ) recovery in the very long pulse. The only fitting parameter employed was the relative fraction of recovered  $U_p$  (about 80%) selected to fix the absolute energy locations of the peaks. The theory used was in existence prior to the first observation of ATI [19]. It is believed that no other theory or TDSE calculation can match this correspondence between theory and experiment in this parameter domain.

the 1980 theory correctly assigned the relative probabilities of the ATI peaks. This result was then extended by Bucksbaum, shown at a 1988 conference [23], to include averaging over the various intensities in the laser focus. Figure 1 shows the result of a recent detailed recalculation, based entirely on the 1980 paper, including averaging over the spatial and temporal variations of laser intensity in the focal region, and also including partial return of ponderomotive energy to the photoelectrons emerging from the very long  $100ps$  pulse duration. The best estimate for peak laser intensity provided by one of the authors [24] of Ref. [20] is employed, and the only adjustable parameter is the relative fraction of  $U_p$  returned to the photoelectron upon leaving the pulse. This parameter establishes the absolute energy

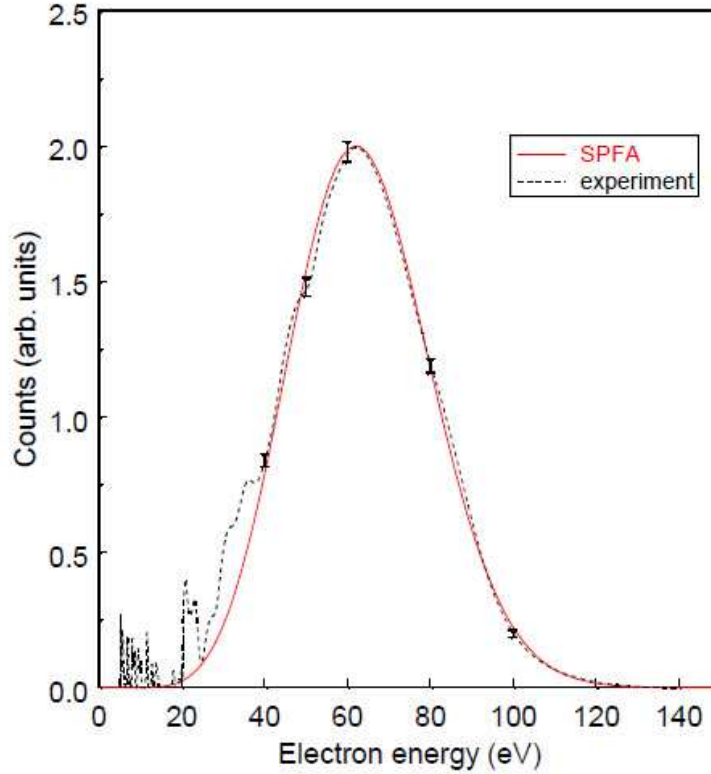


FIG. 2: (color online) This figure, based on Ref. [25], shows the extreme accuracy possible with the transverse-field theory of 1980 [5] applied to the description of an experiment [26] at the relatively high intensity of  $1.27 \times 10^{15} \text{ W/cm}^2$  at  $815 \text{ nm}$  wavelength and a pulse length of  $180 \text{ fs}$  ( $z = 52$ ,  $z_1 = 6.4$ ,  $\gamma_K = 0.40$ ) in the ionization of helium. The fit is within the very small experimental error bars. Irregularities in the low energy part of the spectrum are experimental artifacts [27]. The label “SPFA” stands for “Strong Propagating-Field Approximation” to distinguish it from the ambiguous “SFA”.

location of each of the ATI peaks. These results from a 1986 experiment, described by a 1980 transverse-field theory, have yet to be matched by any proxy-field calculation even 30 years later. Another example of unmatched agreement between experimental measurements and theoretical predictions is with results from a 1993 experiment [26], replicated by a 1996 transverse-field calculation [25], is shown in Fig. 2.

A recent example of the advantages in physical interpretation provided by a transverse-field explanation comes from very precise experiments on the displacement in the propaga-



tion direction of photoelectrons generated by a circularly polarized laser [28]. Performed with nonrelativistic laser fields of wavelengths of  $800nm$  and  $1400nm$ , the very small effect was nevertheless detected and identified by the authors as due to radiation pressure. The results are easily explained qualitatively and quantitatively by a transverse-field theory [29], but the experimenters sought explanation in a DA theory [28, 30, 31]. They experienced daunting problems that have a simple origin: an electric field alone can only provide forces in the polarization direction of the electric field, whereas radiation pressure is exerted in the direction of propagation. The combined action of electric and magnetic fields is necessary to explain a force in the propagation direction. The proxy-field theory has no magnetic component at all.

In summary, laser effects arise from transverse fields while DA proxy fields are longitudinal. Therefore, proxy fields obey completely different Maxwell equations, lose the ability to explain source-free propagation, and therefore provide qualitative explanations that may not be in accord with laboratory reality. The proxy fields require a virtual source current that can inject unphysical energy into a problem if necessary precautions are not observed. Transverse-field strong-field theories [5, 7, 21] have been in existence for a long time, they provide clear physical explanations for strong-laser effects, but further development of these long-standing theories has been neglected in favor of the seemingly simpler dipole-approximation theories that have almost exclusively engaged the attention of strong-field laser research. The defects of the proxy-field approach become severe at very low frequencies, and relief from these difficulties are not to be found in corrections to DA results because the Maxwell equations and the Schrödinger equation involved are inappropriate for the task. Extensions of currently available theories that go beyond the tunneling model can be found from Eqs. (5) - (8), based on simplified versions of the relativistic theories of Refs. [7, 13]. A full appraisal of the applicability of the 1980 theory that goes far beyond the results shown in Figs. 1 and 2 is the subject of a separate manuscript.

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